Review - Pareto Efficiency and Welfare Analysis

Economics 302 - Microeconomic Theory II: Strategic Behavior

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Topics

1. Utility and preferences
2. Pareto efficiency
3. Welfare analysis: consumer and producer surplus

Preference Relation

- **Binary relation** (mathematical relation between two objects) that characterizes an economic agent's preferences on **alternatives/outcomes**:
  - $x \succsim y$: the agent **prefers** x to y;
  - when both $x \succsim y$ and $y \succsim x$, we can write $x \sim y$: the agent is **indifferent** between x and y;
  - when $x \succsim y$ and NOT $y \succsim x$, we can write $x \succ y$: the agent **strictly prefers** x to y - this means that the agent will not choose y if x is available.
- (We will use the words "alternative" and "outcome" interchangeably.)
- An outcome can be anything: a consumption bundle, an amount of money, a quantity of work, the identity of a love partner, etc.
- An outcome can also be a combination of many things.
- What type of outcomes we consider will depend on the economic situation at hand.

What Preferences Can Be Represented by a Utility Function?

- We say that utility function u_i **represents** preference relation $\succsim_i$ when, for any outcomes x and y,

$$u_i(x) \geq u_i(y) \text{ if and only if } x \succsim_i y.$$

- That is: u_i and $\succsim_i$ correspond to the exact same ranking of outcomes (including any ties).
- This is only possible when preferences are **complete** and **transitive**:
  1. Completeness: any two outcomes can be compared by $\succsim_i$.
  2. Transitivity: if $x \succsim_i y$ and $y \succsim_i z$, then $x \succsim_i z$.
- Why?

Non-Uniqueness of Utility Function

- Example: If u_1 represents $\succsim$ and u_2 represents $\succsim'$, what can you say about $2u_1 + 3$ and $0.00001u_2$?
  - Suppose $x \succsim y$. Since u_1 represents $\succsim$, we have $u_1(x) \geq u_1(y)$.
  - Multiply each side by 2 and add 3: $2u_1(x) + 3 \geq 2u_1(y) + 3$.
  - Therefore, the function $2u_1 + 3$ also represents $\succsim$.
  - Similarly, $0.00001u_2$ represents $\succsim'$.
- In general, if u_i represents i's preferences and f is a strictly increasing function, then $f \circ u_i$ also represents i's preferences since $f(u_i(x)) \geq f(u_i(y))$ if and only if $u_i(x) \geq u_i(y)$.
- There are infinitely many increasing functions f. Thus, if a utility function represents a person's preferences, then there are infinitely many utility functions that do so.
- Does it make sense to add or compare utilities of different people?

Quasilinear Utility

- A special case is **quasilinear utility**: a good, called the numeraire (usually money), enters linearly (and positively) into the utility function.
- $u(m, x_1, x_2, ..., x_n) = m + v(x_1, x_2, ..., x_n)$
- Here, m is the numeraire. Its coefficient is usually normalized to 1 (as above) by scaling the utility function.
- If m can be moved (costlessly) across people, and everyone has positive m, then utility can be moved **one-to-one** across people. We then say that utility is **transferable**.
- In this case, it does make sense to compare changes in utility across people.
- Whether it's OK to assume quasilinear utility depends on the economic situation. **You should NOT do this automatically.**

Pareto Efficiency

- When is a situation unambiguously better than another?
- One possible answer: when somebody is better off, and nobody is worse off.
- An outcome is **Pareto dominated** if there exists another outcome where someone is better off, and no one is worse off.
- Mathematically: Suppose the utilities of agents 1 through n are $(u_1, u_2, ..., u_n)$ under outcome a. Then outcome a is Pareto dominated if there exists another outcome b with payoffs $(u_1', u_2', ..., u_n')$, where $u_i' \geq u_i$ for **all** i, **and** $u_j' > u_j$ for **some** j.
    - We then say that b is a **Pareto improvement** over a, or, equivalently, that b **Pareto dominates** a.
- An outcome is **Pareto efficient** if it is not Pareto dominated.
- Caution: being Pareto efficient is **not** the same as Pareto dominating another outcome.

Example and Transferability

- Suppose there are three outcomes giving payoffs $(4,0)$, $(2,1)$ and $(0,3)$.
- If these are the only possible outcomes, which are Pareto efficient?
- Suppose you are instead told that utility is transferable. What other outcomes are there? Are the three outcomes above still Pareto efficient?
- True/false: If an outcome is Pareto efficient, then it maximizes the sum of utilities.
- To do on your own: Prove that if an outcome maximizes the sum of utilities, then it is Pareto efficient, regardless of whether utility is transferable.
- **You should not assume that utility is transferable unless explicitly told.**

Pareto Efficiency in a Market (I)

- Consider a perfectly competitive market with market price P^* and market quantity Q^*.
- As you likely learned in ECON 103, the competitive market outcome is **Pareto efficient**: no Pareto improvement is possible since
  - buyers get all units for which willingness to pay is above P^*, and
  - sellers sell all units for which marginal cost is below P^*, so that
  - all completed transactions are mutually beneficial, and any additional transaction would involve willingness to pay below marginal cost, which means someone would be worse off.
- (In ECON 201, you learned about the First Welfare Theorem, which extends this result to a general equilibrium setting. We will not study general equilibrium in ECON 302.)

Pareto Efficiency in a Market (II)

- Now suppose the price were fixed at P^*:
  - Could the outcome be Pareto efficient if a dictator prevents some transactions and reduces the quantity below Q^*?
  - What if a dictator forces some transactions and increases the quantity above Q^*?
- **There is no such thing as an "efficient price."** The market price is merely the market's way to achieve the **efficient allocation** of goods (which may, however, depend on price if there is an income effect).
- Efficiency $\neq$ Fairness. Economists often focus on the former because, unlike the latter, it does not involve a value judgment: people feel differently about what "fair" means. **However, both are desirable - an efficient outcome may be "worse" than a slightly inefficient, but much fairer outcome.**

Welfare Analysis

- Pareto efficiency is a yes/no welfare criterion. But what if we want to quantify welfare?
- The **social surplus** (SS) in a market (without externalities or taxes/subsidies) is the sum of the producer surplus and the consumer surplus.
  - The **deadweight loss** (DWL) is the amount by which the SS is below the maximum possible SS.
- **Producer surplus** (PS) is the area below the price (P) and above the supply curve, for the units transacted.
- **Consumer surplus** (CS) is the area above the price (P) and below the demand curve, for the units transacted.
- What is so special about the outcome in a competitive market?

What Is Producer Surplus?

- Let the market marginal cost curve be $MC(Q)$. Note that this is the supply curve.
- $PS = PQ - (\text{area under } MC)$
- Denote total revenue by TR, total cost by TC, fixed cost by FC and total variable cost by VC
- Remember that $TR = PQ$ and $VC = \text{area under } MC$.
- $PS = TR - VC = TR - (TC - FC) = (TR - TC) + FC$
- So producer surplus is simply the sum of profit and the fixed cost.

What Is Consumer Surplus? (I)

- Possible answer: net benefit to the consumers, *i.e.* sum of (willingness to pay minus price) for units bought.
- But does it make sense to measure benefit in $ terms? What do consumers maximize?
- CS is an exact concept when utility is quasilinear with $ as the numeraire and agents' bundles include a positive amount of $.
- However, in general, CS converts utility to $ in a sensible but clumsy way. Therefore, it is an **approximate** concept.

What Is Consumer Surplus? (II)

- Suppose that there is a price increase. We know that consumer surplus goes down. What does the change to consumer surplus (ΔCS) measure?
- Consider these questions:
  1. How much money do we need to pay consumers to maintain their utility level? (Compensating variation, or CV)
  2. How much money are consumers willing to pay to avoid the price increase? (Equivalent variation, or EV)
- $CV = EV$ when there is **no income effect**. In this case, $\Delta CS = CV = EV$.
  - Reason: the marginal $ value of the good, which is measured by the demand curve, does not change with the person's "real" income in this case.
- In general, however, CV, EV and ΔCS will be three different quantities.

Link with Quasilinear Utility

- On problem set 1, you will show that there is **no income effect** on any non-numeraire good when utility is **quasilinear** and income is high enough.
- Therefore, in this case, $\Delta CS = CV = EV$.

Why Consumer Surplus?

- When the income effect is small, CV, EV and ΔCS are all close to each other.
- Most goods take up only a small portion of your budget (*e.g.* pencils).
- For these goods, even a large price change will not affect your overall purchasing power too much, and will therefore only have a small income effect.
- Thus, consumer surplus is a sensible approximation most of the time.
- But it's not so great when goods taking up a large part of the budget experience a large price change.

Other Things to Review

- Taxes and subsidies (will use soon)
- Probabilities and expected values (first tutorial)
- Calculus and constrained maximization (substitution and Lagrangian methods)